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Quantum Physics of Nanostructures - Problem Set 2

Winter term 2022/2023

Due date: The problem set will be discussed Friday, 10.11.2022, 13:15-14:45, Room 114.

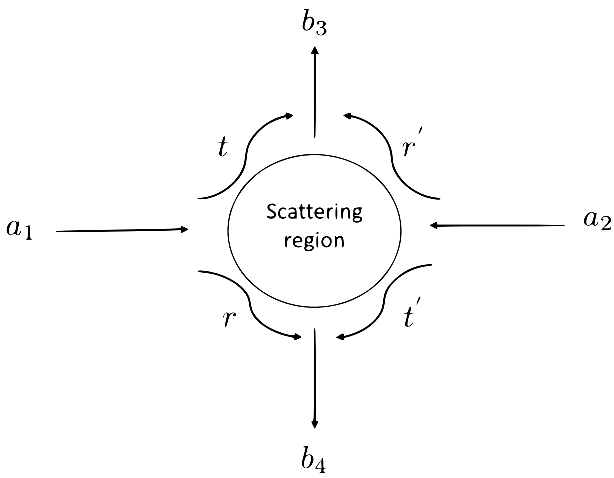
4. Second Quantization formalism of S-matrix *4 + 3 Points*

We consider a single-channel scattering problem, where in the second quantization formalism the output and input states are related via the scattering matrix S like the following

$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \text{ with } S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}.$$

Where $a^\dagger_{1,2}, a_{1,2}$ represent the creation and the annihilation operators of the input states (1: input from left, 2: input from right) which for fermions satisfy the anti-commutation relations (Similarly for the output states creation and the annihilation operators $b^\dagger_{3,4}, b_{3,4}$)

$$\{a_i, a_j^\dagger\} = \delta_{ij}, \quad \{a_i^\dagger, a_j^\dagger\} = 0, \quad \{a_i, a_j\} = 0$$



(a) The number operator is defined as $\hat{n}_i = a_i^\dagger a_i$, show that expectation value of the output number operators are related to the input number operators expectation values via

$$\begin{pmatrix} \langle \hat{n}_3 \rangle \\ \langle \hat{n}_4 \rangle \end{pmatrix} = \begin{pmatrix} R & T' \\ T & R' \end{pmatrix} \begin{pmatrix} \langle \hat{n}_1 \rangle \\ \langle \hat{n}_2 \rangle \end{pmatrix}.$$

with $|r|^2 = R$, $|t|^2 = T$ and $|r'|^2 = R'$, $|t'|^2 = T'$.

(b) Consider a two fermion scattering process with the input state

$$|\Psi_{in}\rangle = a_1^\dagger a_2^\dagger |0\rangle$$

and compute the probability

$$P(1,1) = \langle \Psi_{in} | \hat{n}_3 \hat{n}_4 | \Psi_{in} \rangle$$

5. Current and Noise in 1D

4 Points

In the lectures we have used the operator formalism for the scattering approach and have re-derived the Landauer formula, the aim of this problem is to compute the noise in the system. For a current measured to the left of the scatterer, the current operator is

$$\hat{J}_L(x,\tau) = \frac{e\hbar}{2im}\left[\hat{\psi}_L^\dagger(x,\tau)\,\partial_x\hat{\psi}_L(x,\tau) - \left(\partial_x\hat{\psi}_L^\dagger(x,\tau)\right)\,\hat{\psi}_L(x,\tau)\right] \quad .$$

with the field operator $\hat{\psi}_L$ defined as

$$\hat{\psi}_L(x,\tau) = \frac{1}{2\pi\hbar v}\int d\varepsilon\, e^{-i\varepsilon\tau/\hbar}\left(\hat{a}_\varepsilon e^{ikx} + \hat{a}_\varepsilon e^{-ikx}r + \hat{b}_\varepsilon e^{-ikx}t'\right) \quad .$$

The fluctuation strength of the current is obtained as

$$S(\tau) = \left\langle\left[\hat{J}(x,\tau) - \left\langle\hat{J}(x,\tau)\right\rangle\right]\left[\hat{J}(x,0) - \left\langle\hat{J}(x,\tau)\right\rangle\right]\right\rangle \quad .$$

The noise power is obtained by the Fourier transform

$$S(\omega) = \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau}S(\tau) \quad .$$

Show that the zero frequency noise is given by

$$S(\omega=0) = \frac{e^2}{\pi\hbar}\int d\varepsilon\,\left\{T[f_L(1-f_L) + f_R(1-f_R)] + T(1-T)(f_L-f_R)^2\right\}$$